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# ODAM 2013

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and Applications of Mathematics

Faculty of Science

Palacký Univerzity Olomouc

IT4Innovations
Centre of Excellence

# Intermediate Quantifiers in Fuzzy Natural Logic

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EVROPSKÁ UNIE
EVROPSKÝ FOND PRO REGIONÁLNÍ ROZVOJ
INVESTICE DO VAŠÍ BUDOUCNOSTI

OP Výzkum a vývoj pro inovace

# Outline


1. The concept of natural logic
2. Higher-order fuzzy logic — the main technical tool
3. Evaluative linguistic expressions
4. Quantifiers
5. Intermediate quantifiers
6. Syllogistic reasoning
7. Square of opposition
8. Analysis of the generalized square
9. Conclusions
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George Lakoff: *Linguistic and Natural Logic, Synthese 22, 1970*

## Paradigm of Natural Logic

Natural logic is a collection of terms and rules that come with natural language that allows us to reason and argue in it. Natural language employs a relatively small number of atomic predicates that are related to each other by meaning-postulates that do not vary from language to language.

## Paradigm of FNL

(i) To follow paradigm of natural logic by capturing vagueness phenomenon occurring in its sematics and to develop a working mathematical model of parts of linguistic semantics.

(ii) Develop a mathematical model of natural (commonsense) human reasoning schemes.

**FNL is a mathematical logic extending Mathematical Fuzzy Logic in Narrow Sense (FLn)**

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# Fuzzy Natural Logic

## What FNL can provide?

- Construction of models of systems and processes on the basis of expert knowledge expressed in genuine natural language
- Making computer to “understand” natural language and behave accordingly

## Current constituents of FNL

- Theory of evaluative linguistic expressions (*small*, *very small*, *medium*, *large*, etc.)
- Theory of fuzzy/linguistic IF-THEN rules and logical inference (Perception-based Logical Deduction)
- **Theory of intermediate quantifiers** ( *most*, *a lot of*, *few*, *many*, etc.) and **generalized (intermediate) Aristotle syllogisms**

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# Higher-order fuzzy logic — Fuzzy Type Theory

Logical analysis of concepts and natural language expressions requires higher-order logic — **type theory**.

**Why fuzzy type theory**

- It is a constituent of Mathematical Fuzzy Logic, well established with good mathematical properties.
- Makes it possible to include model of vagueness in the developed mathematical models

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# Fuzzy Type Theory

Generalization of classical **type theory**

- *Founder:* B. Russel (1903, 1908)
- A. Church, L. Henkin, P. Andrews, P. Martin-Löf

Syntax of FTT is an extended lambda calculus:

- more logical axioms
- many-valued semantics

**Main fuzzy type theories**

IMTL, Łukasiewicz, EQ-algebra based

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# Truth values in FTT

*Standard Łukasiewicz MV$_\Delta$-algebra*

$$\mathscr{E} = \langle [0,1], \vee, \wedge, \otimes, \Delta, \rightarrow, 0, 1\rangle$$

$$\vee, \wedge = \text{minimum, maximum}$$

$$a \otimes b = 0 \vee (a + b - 1) \qquad \text{(Łukasiewicz conjunction)}$$

$$a \rightarrow b = 1 \wedge (1 - a + b) \qquad \text{(Łukasiewicz implication)}$$

$$\neg a = a \rightarrow 0 = 1 - a \qquad \neg\neg a = a$$

$$a \leftrightarrow b = (a \rightarrow b) \wedge (b \rightarrow a) \qquad \Delta(a) = \begin{cases} 1 & \text{if } a = 1, \\ 0 & \text{otherwise.} \end{cases}$$

# Interpretation of formulas

**Example**

$\mathscr{M}^{\mathcal{E}}(A_o) \in E$ is a truth value

$\mathscr{M}^{\mathcal{E}}(A_\alpha \equiv B_\alpha) \in E$ is a truth degree of fuzzy equality between $\mathscr{M}^{\mathcal{E}}(A_\alpha)$ and $\mathscr{M}^{\mathcal{E}}(B_\alpha)$

$\mathscr{M}^{\mathcal{E}}(A_{o\epsilon}) : M_\alpha \longrightarrow E$ is a fuzzy set in $M_\epsilon$

$\mathscr{M}^{\mathcal{E}}(A_{(o\epsilon)\epsilon}) : M_\epsilon \longrightarrow M_o^{M_\epsilon}$ is a fuzzy relation on $M_\epsilon$

$\mathscr{M}^{\mathcal{E}}(A_{\epsilon\epsilon}) : M_\epsilon \longrightarrow M_\epsilon$ is a function on objects

**Generalized completeness holds for FTT:**

$$T \vdash A_o \quad \textbf{iff} \quad T \models A_o$$

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# Evaluative linguistic expressions

**Pure evaluative expressions:**

*very short, rather strong, more or less medium, roughly big, extremely big*

## Components

(i) **TE-adjectives:**

*small, medium, big; weak, medium strong, strong; silly, normal, intelligent; good, average, bad*

(ii) **Hedges:**

- Narrowing: *very, extremely, significantly*
- Widening: *more or less, roughly, very roughly*
- Specifying: *approximately, about, rather, precisely, typically*

**Other evaluative expressions:**

Fuzzy numbers, compound, negative

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# Semantics of natural language in FNL

Evaluative linguistic expressions characterize part of a bounded ordered scale in a certain **context**

Possible world = **Context**

$$w = \langle v_L, v_S, v_R \rangle, \qquad v_L, v_S, v_R \in \mathbb{R}$$

*Distance:*

$w = \langle 10, 100, 400 \rangle$ *(Czech Republic)*
$w = \langle 100, 500, 3\ 000 \rangle$ *(Russia)*

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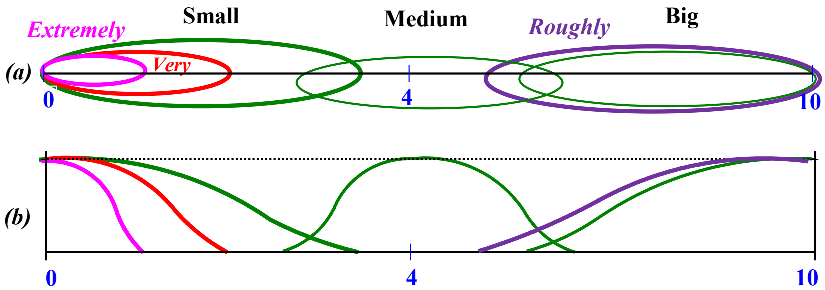
# Extensions of evaluative expressions

Context $w = \langle 0, 4, 10 \rangle$




Semantics of evaluative linguistic expressions: Special theory $T^{\mathrm{Ev}}$ of ŁFTT

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# Classical and generalized quantifiers

**(i)** Classical quantifiers
Aristotelian logic — the quantifiers *All, Some, No*;
G. Frege, Ch. S. Pierce, O. H. Mitchell— the quantifiers $\forall, \exists$

**(ii)** Generalized quantifiers
A. Mostowski, P. Lindström, J. van Benthem, J. Barwise, R. Cooper, L. E. Keenan, D. Westerståhl
*Most, at least five, many, an odd number of*, etc.

**(iii)** Fuzzy quantifiers
L. A. Zadeh,
Generalization of (ii): I. Glöckner (semi-fuzzy quantifiers), M. Holčapek, A. Dvořák
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# Intermediate quantifiers

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**Quantifiers in natural language**

Words (expressions) that precede and modify nouns; tell us how many or how much. They specify quantity of specimens in the domain of discourse having a certain property.

**Example**

*All, Most, Almost all, Few, Many, Some, No*

Most women in the party are well dressed

Few students passed exam

*Important subclass of generalized quantifiers*

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# Semantics of intermediate quantifiers

**Main idea**

Intermediate quantifiers are classical quantifiers $\forall$ and $\exists$ taken over a *smaller* class of elements. Its size is determined using an appropriate evaluative expression.

Classical logic: *No substantiation why and how the range of quantification should be made smaller*

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Classical logic: *No substantiation why and how the range of quantification should be made smaller*

# Formal theory of intermediate quantifiers

**"Most $B$'s are $A$" ("Most small children are slim")**

$$(Q^{\forall}_{Bi\,Ve}\,x)(B,A) := \underbrace{(\exists z)((\mathbf{\Delta}(z \subseteq B)}_{\text{“the greatest” fuzzy subset of } B} \quad \& \quad \underbrace{(\forall x)(z\,x \Rightarrow Ax))}_{\text{each of } z\text{'s has } A} \quad \wedge \quad \underbrace{Bi\,Ve((\mu B)z))}_{\text{size of } z \text{ is evaluated as } Very\ big}$$

# Formal theory of intermediate quantifiers

$T^{\mathrm{IQ}} = T^{\mathrm{Ev}} +$ 4 special axioms

**"⟨Quantifier⟩ $B$'s are $A$"**

$$(Q^{\forall}_{Ev}\, x)(B, A) := \underbrace{(\exists z)((\boldsymbol{\Delta}(z \subseteq B)}_{\text{"the greatest" fuzzy subset of } B\text{'s}} \;\; \& \;\; \underbrace{(\forall x)(z\,x \Rightarrow Ax))}_{\text{each of } z\text{'s has } A} \;\; \wedge \;\; \underbrace{Ev((\mu B)z))}_{\text{size of } z \text{ is evaluated by } Ev}$$

$Ev$ — extension of a certain evaluative expression

(*big, very big, small*, etc.)

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# Special intermediate quantifiers

*Classical quantifiers*

**A:** All $B$ are $A := (Q^{\forall}_{Bi\boldsymbol{\Delta}}x)(B, A) \equiv (\forall x)(Bx \Rightarrow Ax)$,

**E:** No $B$ are $A := (Q^{\forall}_{Bi\boldsymbol{\Delta}}x)(B, \neg A) \equiv (\forall x)(Bx \Rightarrow \neg Ax)$,

**I:** Some $B$ are $A := (Q^{\exists}_{Bi\boldsymbol{\Delta}}x)(B, A) \equiv (\exists x)(Bx \wedge Ax)$,

**O:** Some $B$ are not $A := (Q^{\exists}_{Bi\boldsymbol{\Delta}}x)(B, \neg A) \equiv (\exists x)(Bx \wedge \neg Ax)$.

# Special intermediate quantifiers

**P:** Almost all $B$ are $A$ := $(Q^{\forall}_{Bi\,Ex}x)(B, A)$

$\equiv$

$$(\exists z)((\boldsymbol{\Delta}(z \subseteq B)\,\&\,(\forall x)(zx \Rightarrow Ax)) \wedge (Bi\,Ex)((\mu B)z)),$$

**B:** Few $B$ are $A$ (:= Almost all $B$ are not $A$) := $(Q^{\forall}_{Bi\,Ex}x)(B, \neg A)$

$\equiv$

$$(\exists z)((\boldsymbol{\Delta}(z \subseteq B)\,\&\,(\forall x)(zx \Rightarrow \neg Ax)) \wedge (Bi\,Ex)((\mu B)z)),$$

**T:** Most $B$ are $A$ := $(Q^{\forall}_{Bi\,Ve}x)(B, A)$

$\equiv$

$$(\exists z)((\boldsymbol{\Delta}(z \subseteq B)\,\&\,(\forall x)(zx \Rightarrow Ax)) \wedge (Bi\,Ve)((\mu B)z)),$$

**D:** Most $B$ are not $A$ := $(Q^{\forall}_{Bi\,Ve}x)(B, \neg A)$

**K:** Many $B$ are $A$ := $(Q^{\forall}_{\neg(Sm\,\bar{\boldsymbol{\nu}})}x)(B, A)$

**G:** Many $B$ are not $A$ := $(Q^{\forall}_{\neg(Sm\,\bar{\boldsymbol{\nu}})}x)(B, \neg A)$

# Special intermediate quantifiers

Our theory enables to develop mathematical model of the meaning of complicated expressions of natural language including their vagueness

**Example**

- Almost all birds are good flyers.
- Many people in the party are women.
- Most school children are picky eaters.
- Few dresses are yellow.

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# Generalized (intermediate) syllogisms

**Definition**

- A syllogism $\langle P_1, P_2, C\rangle$ — logical argument in which the *conclusion* $C$ is inferred from two *premises* — *major* $P_1$ and *minor* $P_2$.
  - Major premise ($P_1$): All humans are mortal.
  - Minor premise ($P_2$): Some animals are human.
  - Conclusion ($C$): Some animals are mortal.
- Intermediate syllogism: traditional syllogism in which some classical quantifiers are replaced by intermediate ones.
- We say that the syllogism is (strongly) valid (in $T^{\mathrm{IQ}}$) if

$$T^{\mathrm{IQ}} \vdash P_1 \,\&\, P_2 \Rightarrow C,$$

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# 105 valid generalized Aristotle's syllogisms

**Figure I**

$$\frac{\begin{array}{c} Q_1\ M \text{ is } Y \\ Q_2\ X \text{ is } M \end{array}}{Q_3\ X \text{ is } Y}$$

**Figure II**

$$\frac{\begin{array}{c} Q_1\ Y \text{ is } M \\ Q_2\ X \text{ is } M \end{array}}{Q_3\ X \text{ is } Y}$$

**Example (ATT-I)**

All women are well dressed
Most people in the party are women
---
Most people in the party are well dressed

**Example (ETO-II)**

No lazy people pass exam
Most students pass exam
---
Some students are not lazy people

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**Figure III**

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**Example (PPI-III )**

Almost all old people are ill
Almost all old people have gray hair
———
Some people with gray hair are ill

**Example (TAI-IV )**

*Most shares with downward trend are from car industry
All shares of car industry are important
———
Some important shares have downward trend

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**Figure III**

$$\frac{\begin{array}{c} Q_1\ M \text{ is } Y \\ Q_2\ M \text{ is } X \end{array}}{Q_3\ X \text{ is } Y}$$

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|---|---|
| $Q_1$ $M$ is $Y$ | $Q_1$ $Y$ is $M$ |
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| $Q_3$ $X$ is $Y$ | $Q_3$ $X$ is $Y$ |

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**Example (TAI-IV )**

*Most shares with downward trend are from car industry
All shares of car industry are important
---
Some important shares have downward trend

# Outline

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# Classical relations between formulas

**Definition (Classical logic)**

- Two formulas $P_1, P_2$ are contradictory if in any model they cannot both be true, and they cannot both be false.
- Two formulas $P_1, P_2$ are contraries if in any model they cannot both be true, but both can be false.
- Two formulas $P_1, P_2$ are sub-contraries if in any model they cannot both be false, but both can be true.
- A formula $P_1$ is subaltern of $P_2$ (superaltern) if, in any model, it must be true if its superaltern is true. The superaltern must be false if the subaltern is false.

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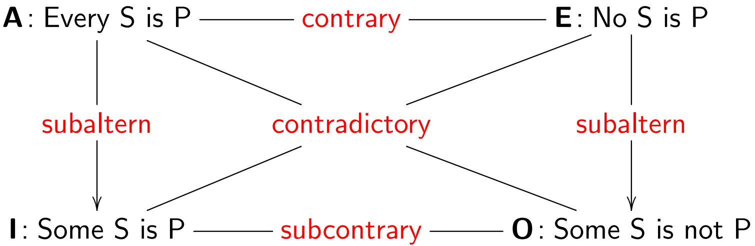
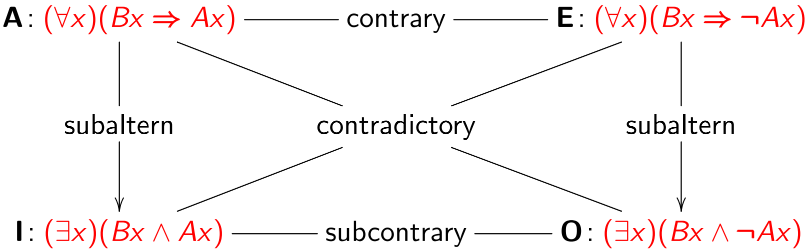
# Aristotle square of opposition



# Aristotle square with logical formulas



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# Generalized definitions

*Contraries (classical definition)*

- $P_1, P_2$ are contraries if in any model they cannot be both true but can be both false.

*Contraries (generalized definition)*

- $P_1, P_2$ are contraries in $T^{\mathrm{IQ}}$ if $T^{\mathrm{IQ}} \vdash P_1 \,\&\, P_2 \equiv \bot$.

$$\mathscr{M}(P_1) \otimes \mathscr{P} = \max(0, \mathscr{M}(P_1) + \mathscr{M}(P_2) - 1)$$

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# Generalized definitions

*Sub-contraries (classical definition)*

- $P_1, P_2$ are sub-contraries if in any model they cannot be both false but can be both true.

*Sub-contraries (generalized definition)*

- $P_1$ and $P_2$ are sub-contraries in $T^{\mathrm{IQ}}$ if $T^{\mathrm{IQ}} \vdash P_1 \nabla P_2 \not\equiv \bot$

$\mathscr{M}(P_1) \oplus \mathscr{P} = \min(1, \mathscr{M}(P_1) + \mathscr{M}(P_2))$

# Generalized definitions

*Contradictory (classical definition)*

- $P_1, P_2$ are contradictory if in any model they cannot be both true and they cannot be both false.

*Contradictory (generalized definition)*

- $P_1$ and $P_2$ are contradictory in $T^{IQ}$ if $T^{IQ} \vdash \boldsymbol{\Delta} P_1 \,\&\, \boldsymbol{\Delta} P_2 \equiv \bot$ as well as $T^{IQ} \vdash \boldsymbol{\Delta} P_1 \nabla \boldsymbol{\Delta} P_2$.

$$\mathscr{M}(P_1) = 1, \mathscr{M}(P_2) < 1 \qquad \text{or} \qquad \mathscr{M}(P_1) < 1, \mathscr{M}(P_2) = 1$$

# Generalized definitions

**Sub-altern (classical definition)**

- $P_2$ is subaltern of $P_1$ called superaltern if, in any model, it must be true if its superaltern is true. At the same time, the superaltern must be false if the subaltern is false.

**Sub-altern (generalized definition)**

- $P_2$ is sub-altern of $P_1$ in $T^{\mathrm{IQ}}$ if $T^{\mathrm{IQ}} \vdash P_1 \Rightarrow P_2$.

$\mathscr{M}(P_1) \leq \mathscr{M}(P_2)$

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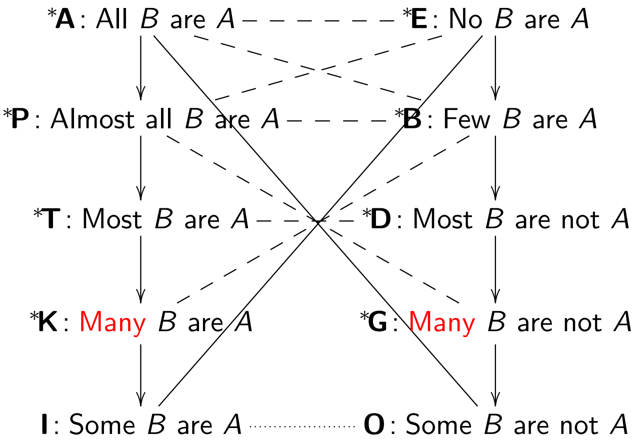
# *Generalized Peterson's square*



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# Conclusions

- We presented the concept of fuzzy natural logic — basic tool: higher-order fuzzy logic (FTT).
- We demonstrated how semantics of special classes of natural language expressions including their vagueness can be formalized
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- Holčapek, M., *Monadic* **L**-*fuzzy quantifiers of the type* $\langle 1^n, 1\rangle$, Fuzzy Sets and Systems 159(2008), 1811-1835.
- Dvořák, A. and Holčapek, M., *L-fuzzy Quantifiers of the Type* $\langle 1\rangle$ *Determined by Measures*, Fuzzy Sets and Systems, 160(2009), 3425-3452
- V. Novák, *On fuzzy type theory*, Fuzzy Sets and Systems 149 (2005), 235–273.
- V. Novák, *EQ-algebra-based fuzzy type theory and its extensions*, Logic Journal of the IGPL 19(2011), 512-542
- V. Novák, *A comprehensive theory of trichotomous evaluative linguistic expressions*, Fuzzy Sets and Systems 159(2008), 2939—2969.
- V. Novák, *A formal theory of intermediate quantifiers, Fuzzy Sets and Systems* 159(2008) 1229–1246
- P. Murinová, V. Novák, *A formal theory of generalized intermediate syllogisms*, Fuzzy Sets and Systems 186(2012), 47-80
- P. L. Peterson, Intermediate Quantifiers. Logic, linguistics, and Aristotelian semantics, Ashgate, Aldershot 2000.