european
social fund in the
czech republic

EUROPEAN UNION

MINISTRY OF EDUCATION,
YOUTH AND SPORTS

OP Education
for Competitiveness

INVESTMENTS
IN EDUCATION
DEVELOPMENT

**Streamlining the Applied Mathematics Studies
at Faculty of Science of Palacký University in Olomouc
CZ.1.07/2.2.00/15.0243**

International Conference
Olomoucian Days of Applied Mathematics

ODAM 2011

Department of Mathematical analysis
and Applications of Mathematics
Faculty of Science
Palacký University Olomouc

An introduction to principles of collective decisions

Milan Vlach

Faculty of Mathematics and Physics, Charles University
Malostranské náměstí 25, 118 00 Praha 1, Czech Republic
e-mail: milan.vlach@mfff.cuni.cz

Kyoto College of Graduate Studies for Informatics

Problem Formulation

We shall be concerned with problems encountered by a group of individuals (agents, persons, players, voters, criteria, etc.) who wish to select one alternative (issues, candidates, strategies, outcomes, objects, etc.) from a given set of alternatives in such a way that the possibly conflicting individual preferences are fairly taken into account.

Such problems were aptly characterized with Amartya Sen's comment (in his 1998 Nobel Prize lecture) that

a camel is a horse designed by a committee.

But the deficiencies of committee decisions can be even worse because a committee that tries to reflect the diverse wishes of its different members in designing a horse could very easily end up with something far less coherent: perhaps a centaur of Greek mythology, half a horse and half something else.

One possibility of resolving potential conflict among individual preferences is to use some reasonable procedure for aggregation of individual preferences into preferences of the group, and then to select the best alternative with respect to the resulting group preferences.

To illustrate some of the obstacles and difficulties that should be resolved, we first consider simple examples.

1	2	3
a	c	a
b	b	c
c	a	b

1	2	3
a	c	b
b	b	a
c	a	c

1	2	3
a	b	c
b	c	a
c	a	b

1	2	3
a	c	a
b	b	c
c	a	b

$\longmapsto$

Group
a
b

1	2	3
a	c	b
b	b	a
c	a	c

1	2	3
a	b	c
b	c	a
c	a	b

1	2	3
a	c	a
b	b	c
c	a	b

$\longmapsto$

Group	
a	c
b	b

1	2	3
a	c	b
b	b	a
c	a	c

1	2	3
a	b	c
b	c	a
c	a	b

1	2	3
a	c	a
b	b	c
c	a	b

$\longmapsto$

Group		
a	c	a
b	b	c

1	2	3
a	c	b
b	b	a
c	a	c

1	2	3
a	b	c
b	c	a
c	a	b

1	2	3
a	c	a
b	b	c
c	a	b

$\longmapsto$

Group		
a	c	a
b	b	c

$\longmapsto$

Group
a
c
b

1	2	3
a	c	b
b	b	a
c	a	c

1	2	3
a	b	c
b	c	a
c	a	b

1	2	3
a	c	a
b	b	c
c	a	b

$$\longmapsto$$

Group		
a	c	a
b	b	c

$$\longmapsto$$

Group
a
c
b

1	2	3
a	c	b
b	b	a
c	a	c

$$\longmapsto$$

Group		
b	b	a
a	c	c

1	2	3
a	b	c
b	c	a
c	a	b

1	2	3
a	c	a
b	b	c
c	a	b

$\longmapsto$

Group		
a	c	a
b	b	c

$\longmapsto$

Group
a
c
b

1	2	3
a	c	b
b	b	a
c	a	c

$\longmapsto$

Group		
b	b	a
a	c	c

$\longmapsto$

Group
b
a
c

1	2	3
a	b	c
b	c	a
c	a	b

$$\begin{array}{ccc}1 & 2 & 3\\\hline \mathsf{a} & \mathsf{c} & \mathsf{a}\\ \mathsf{b} & \mathsf{b} & \mathsf{c}\\ \mathsf{c} & \mathsf{a} & \mathsf{b}\end{array} \longmapsto \begin{array}{ccc}\multicolumn{3}{c}{\text{Group}}\\\hline \mathsf{a} & \mathsf{c} & \mathsf{a}\\ \mathsf{b} & \mathsf{b} & \mathsf{c}\end{array} \longmapsto \begin{array}{c}\text{Group}\\\hline \mathsf{a}\\ \mathsf{c}\\ \mathsf{b}\end{array}$$

$$\begin{array}{ccc}1 & 2 & 3\\\hline \mathsf{a} & \mathsf{c} & \mathsf{b}\\ \mathsf{b} & \mathsf{b} & \mathsf{a}\\ \mathsf{c} & \mathsf{a} & \mathsf{c}\end{array} \longmapsto \begin{array}{ccc}\multicolumn{3}{c}{\text{Group}}\\\hline \mathsf{b} & \mathsf{b} & \mathsf{a}\\ \mathsf{a} & \mathsf{c} & \mathsf{c}\end{array} \longmapsto \begin{array}{c}\text{Group}\\\hline \mathsf{b}\\ \mathsf{a}\\ \mathsf{c}\end{array}$$

$$\begin{array}{ccc}1 & 2 & 3\\\hline \mathsf{a} & \mathsf{b} & \mathsf{c}\\ \mathsf{b} & \mathsf{c} & \mathsf{a}\\ \mathsf{c} & \mathsf{a} & \mathsf{b}\end{array} \longmapsto \begin{array}{ccc}\multicolumn{3}{c}{\text{Group}}\\\hline \mathsf{a} & \mathsf{b} & \mathsf{c}\\ \mathsf{b} & \mathsf{c} & \mathsf{a}\end{array}$$

1	2	3
a	c	a
b	b	c
c	a	b

$\longmapsto$

Group		
a	c	a
b	b	c

$\longmapsto$

Group
a
c
b

1	2	3
a	c	b
b	b	a
c	a	c

$\longmapsto$

Group		
b	b	a
a	c	c

$\longmapsto$

Group
b
a
c

1	2	3
a	b	c
b	c	a
c	a	b

$\longmapsto$

Group		
a	b	c
b	c	a

$\longmapsto$

Group
?
?
?

We now have rather unpleasant situation. Each individual preference relation is without any contradiction, but the resulting group preference relation

a *is better than* b, b *is better than* c, c *is better than* a

is self-contradictory; it is cyclic and, consequently, no alternative is best for the group. In other words, each member of the group was rational, but the group became rather irrational (unreasonable, illogical, inconsistent).

This phenomenon, which may occur in application of majority voting, is known as the *voting paradox* or *Condorcet's effect*.

Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet (17 September 1743 28 March 1794), known as Nicolas de Condorcet, was a French philosopher, mathematician, and early political scientist whose Condorcet method in voting tally selects the candidate who would beat each of the other candidates in a run-off election.

Unlike many of his contemporaries, he advocated a liberal economy, free and equal public education, constitutionalism, and equal rights for women and people of all races. His ideas and writings were said to embody the ideals of the Age of Enlightenment and rationalism, and remain influential to this day.

He died a mysterious death in prison after a period of being a fugitive from French Revolutionary authorities.

Arnold B. Urken (Electronic Journ@l for History of Probability and Statistics, Vol 4. June 2008):

Condorcets Essai sur l'application de l'analyse la probabilit des dcisions rendues la pluralit des voix [Condorcet, 1785] is one of the most frequently cited, least-read, and poorly-understood works in voting theory.

Scholars seem to agree that the Essai is a classic that must be cited even though the broad scope of its argument and its 495 pages are not easily accessible to modern readers.

So it is not surprising that the Marquis ideas are sometimes developed without taking account of a precise or accurate contextual appreciation of his arguments.

We have demonstrated that the majority voting may aggregate a sequence (profile) of *complete asymmetric transitive* relations into an intransitive relation, even in a very simple situation involving only three alternatives and three voters. In this connection, some natural questions arise. For example:

- Is such an undesirable property typical for the majority voting?

- Is it possible to guarantee that this effect will not occur in particular cases?

- Are there reasonable aggregation procedures without this drawback?

In our particular case of three individuals and three alternatives, we have 216 profiles

1	2	3
.	.	.
.	.	.
.	.	.

Among these 216 profiles there are twelve which give rise to the Condorcet effect; that is, a little less than 6 percent

This proportion increases with the number of voters; for example:

5 voters . . . 7 percent
9 voters . . . 7.8 percent.

The following table gives more detailed information to the first question:

$m \backslash n$	3	5	7	9	...	limit
3	.056	.069	.075	.078	...	.088
4	.111	.139	.150	.156	...	.176
5	.160	.200	.215	.230	...	.251
6	.202	.255	.258	.284	...	.315
7	.239	.299	.305	.342	...	.369
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$
limit	1.000	1.000	1.000	1.000	...	1.000

Each entry of the table presents the approximate fractions of the set of all n-tuples of linear orders on the set of m alternatives for which the simple majority rule gives an intransitive relation.

One way of avoiding the paradox is to limit the freedom of players to choose arbitrary preferences. If we accept this idea, then it is desirable to limit the freedom of players as less as possible. This leads to an interesting problem, whose many aspects are still waiting for a satisfactory solution.

Let us assume that the players are never indifferent between distinct alternatives. Then their preferences can be represented by permutations of alternatives. In the case of three alternatives x, y and z, the preference relation of a player is represented by one of the six possible permutations of alternatives

$$xyz,\ xzy,\ yxz,\ yzx,\ zxy,\ zyx.$$

Let us forbid one of the orders, say xyz. Does it help? Not really, as we can see from the following preferences. Suppose that

player 1 prefers y to x, x to z, and y to z,
player 2 prefers x to z, z to y, and x to y,
player 3 prefers z to y, y to x, and z to x.

Then, in the simple majority voting, y wins over x, x wins over z, and z wins over y.

Because similar results hold also for other orders, we have to forbid more than one order to avoid the paradox.

It can easily be verified that we must exclude at least one of the orders

$$xyz, \; zxy, \; yzx$$

and also at least one of the orders

$$zyx, \; xzy, \; yxz.$$

It turns out that these necessary conditions are also sufficient in the case of three alternatives. As a consequence, for example, the set consisting of the folowing four orders

$$xyz, \; zxy, \; xzy, \; yxz$$

has the property that (independently of how many players we have) if the preference relation of each player belongs to this set, then the paradox is avoided. Let us call such sets of orders acyclic sets.

More formally: Let S_m denote the set of linear orders (permutations) on $\{1, 2, \ldots, m\}$. A subset T of S_m is *a*cyclic if there exists no set of individuals with preference orders in T whose preferences induce a majority cycle on three or more members of $\{1, 2, \ldots, m\}$. The problem is to determine

$$f(m) = \text{max}\{|T| : T \text{ is an acyclic subset of } S_m\}$$

for each positive integer m, and to describe the structure of the maximum acyclic subsets of S_m.

For example, it is known that the largest acyclic sets for four alternatives consists of nine orders, and the largest acyclic sets for five alternatives consists of twenty orders. Also it is known that

$$f(m) \geq 2^{m-1} + 2^{m-3} - 1.$$

Peter C. Fishburn. *Decision theory and discrete mathematics*. Discrete Applied Mathematics 68 (1996), 209-221.

Peter C. Fishburn. *Acyclic sets of linear orders: A progress report*. Social Choice and Welfare, 19 (2002), 431-447, 2002.

Bernard Monjardet. *Acyclic domains of linear orders: a survey*. Cahiers de la Maison des Sciences Economiques b06083, 2006, Université Panthéon-Sorbonne (Paris 1).

1	2	3
a	b	c
b	c	a
c	a	b

$\longmapsto$

Group		
a	b	c
b	c	a

$\longmapsto$

Group
?
?
?

1	2	3
.	.	.
.	.	.
.	.	.

$\longmapsto$

Group
.
.
.

Let X be a fixed set of alternatives having at least three elements, let N be a fixed finite set of individuals with at least two members, and let $\mathcal{B}$ be the set of all binary relations on X.

A choice set with respect to subset A of X and a relation R from $\mathcal{B}$, denoted by $C(A, R)$, is the set of those alternatives x from A for which xRy for all y from A.

A relation $R \in \mathcal{B}$ is *choice relation* if $C(A, R)$ is nonempty for every finite subset A of X. The set of choice relations is denoted by $\mathcal{C}$.

The set of all non-strict linear orders on X is denoted by $\mathcal{R}$, and the set of ordered n-tuples of relations from $\mathcal{R}$ is denoted by $\mathcal{R}^n$.

Similarly, the set of all strict linear orders on X is denoted by $\mathcal{P}$, and the set of ordered n-tuples of relations from $\mathcal{P}$ is denoted by $\mathcal{P}^n$.

A collective choice rule is a mapping from $\mathcal{R}^n$ into $\mathcal{B}$.

A social welfare function is a collective choice rule, the range of which is restricted to $\mathcal{R}$.

A social decision function is a collective choice rule, the range of which is restricted to $\mathcal{C}$.

Condition 1. *(Weak Pareto efficiency)* A social welfare function or a social decision function is *weakly Pareto efficient* if, for all a and b from X, we have aPb whenever aP_ib for each individual $i \in N$.

Condition 2. *(Non-dictatorship)* A social welfare function is *non-dictatorial* if no individual $i \in N$ has the property: for all alternatives $a, b \in X$ and each profile $(R_1, R_2, \ldots, R_n)$, we have aRb whenever aR_ib.

Condition 3. *(Independence)* A social welfare function is *independent from irrelevant alternatives* if the social preference between any two alternatives depends only on the individual preferences between those two alternatives.

Theorem (Arrow). There is no social welfare function $F : \mathcal{P}^n \to \mathcal{P}$ simultaneously satisfying

- Pareto condition,
- Non-dictatorship, and
- Independence from irrelevant alternatives.

A social decision function is called *liberal* if, for each individual, there is at least one pair of distinct alternatives, say x and y, such that i is decisive over that pair of alternatives; that is if i prefers x to y, then society must do the same; and if i prefers y to x then society has to choose this preference.

A social decision function is called *minimal liberal* if there are at least two distinct individuals i and j such that each of them is decisive over at least one pair of alternatives, say $x, y, x \neq y$ and $z, w, z \neq w$.

A social decision function is called *super-minimal liberal* if there are at least two distinct individuals i and j such that each of them is semi-decisive over at least one pair of distinct alternatives, say x, y and z, w, with $x \neq z$ and $y \neq w$.

Theorem (Sen). There is no social decision function that is simultaneously weakly Pareto efficient and super-minimal liberal.

Corollary (Sen). There is no social decision function that is simultaneously weakly Pareto efficient and minimal liberal.

Theorem (Sen). There does not exist a social decision function that ranks the alternatives and always satisfies the following conditions:

- Minimal Liberalism: There are at least two agents each of whom is decisive over at least one assigned pair of alternatives. Their ranking of the assigned pairs of alternatives determine the societal ranking of the pairs.

- Weak Pareto Efficiency: If for any pair of alternatives, all voters rank the pair in the same manner, then this unanimous ranking is the societal ranking of the pair.

- Acyclic: The outcome does not have any cycles.

K. J. Arrow: *Social Choice and Individual Values*, 2nd. ed., Wiley, New York, 1963.

A. K. Sen: *Collective Choice and Social Welfare*, San Francisco: Holden-Day; and Edinburgh: Oliver & Boyd, 1970.

J. S. Kelly: *Social Choice Theory: An introduction*, Berlin, Springer, 1988.

K. J. Arrow, A. K. Sen and K. Suzumura, eds.: *Handbook of Social Choice and Welfare*, Elsevier, Amsterdam, Volume 1, 2002.

Appendix

A *social choice procedure* for a set X of alternatives and a set of individuals is a mapping that assigns to every sequence (profile) of the individual preference relations on X a nonempty subset A of X, which we call the *social choice set*.

To illustrate various social choice procedures, we shall use the following profile of individual preferences:

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

Plurality Voting

The social choice set is the set of alternatives with the largest number of the first-place rankings in the individual preference lists.

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

Obviously, the social choice set is $\{a\}$.

The Borda Count

The alternative at the bottom of the preference list gets zero points, the alternative at the next to bottom spot gets one point, the next one up gets two points, and so on up to the top alternative on the list. Then, for each alternative, the points awarded it are added up. The social choice set is the set of alternatives with the highest sum.

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

$A = \{?\}$

The Borda Count

The alternative at the bottom of the preference list gets zero points, the alternative at the next to bottom spot gets one point, the next one up gets two points, and so on up to the top alternative on the list. Then, for each alternative, the points awarded it are added up. The social choice set is the set of alternatives with the highest sum.

V 1	V 2	V 3	V 4	V 5	V 6	V 7	Points
a	a	a	c	c	b	e	4
b	d	d	b	d	c	c	3
c	b	b	d	b	d	d	2
d	e	e	e	a	a	b	1
e	c	c	a	e	e	a	0

$A = \{?\}$

The Borda Count

The alternative at the bottom of the preference list gets zero points, the alternative at the next to bottom spot gets one point, the next one up gets two points, and so on up to the top alternative on the list. Then, for each alternative, the points awarded it are added up. The social choice set is the set of alternatives with the highest sum.

V 1	V 2	V 3	V 4	V 5	V 6	V 7	Points
a	a	a	c	c	b	e	4
b	d	d	b	d	c	c	3
c	b	b	d	b	d	d	2
d	e	e	e	a	a	b	1
e	c	c	a	e	e	a	0

$A = \{?\} = \{b\}$

V 1	V 2	V 3	V 4	V 5	V 6	V 7	Points
a	a	a	c	c	b	e	4
b	d	d	b	d	c	c	3
c	b	b	d	b	d	d	2
d	e	e	e	a	a	b	1
e	c	c	a	e	e	a	0

$$\begin{aligned} a: 4+4+4+0+1+1+0 &= 14 \\ b: 3+2+2+3+2+4+1 &= 17 \\ c: 2+0+0+4+4+3+3 &= 16 \\ d: 1+3+3+2+3+2+2 &= 16 \\ e: 0+1+1+1+0+0+4 &= 7 \end{aligned}$$

The Hare System

The alternative or alternatives occurring at the top of at least half of the individual preference lists form the social choice set. If no alternative occurs at the top of at least half of the preference lists, then the alternatives occurring at the top of the fewest lists are deleted from all preference lists and the process of seeking alternatives on the top of at least half of the lists is repeated, and so on.

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

The Hare System

The alternative or alternatives occurring at the top of at least half of the individual preference lists form the social choice set. If no alternative occurs at the top of at least half of the preference lists, then the alternatives occurring at the top of the fewest lists are deleted from all preference lists and the process of seeking alternatives on the top of at least half of the lists is repeated, and so on.

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

$$A = \{c\}$$

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

V 1	V 2	V 3	V 4	V 5	V 6	V 7

V 1	V 2	V 3	V 4	V 5	V 6	V 7

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	b	b	b	b	c	c
c	e	e	e	a	a	b
e	c	c	a	e	e	a

V 1	V 2	V 3	V 4	V 5	V 6	V 7

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	b	b	b	b	c	c
c	e	e	e	a	a	b
e	c	c	a	e	e	a

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	c	c
c	c	c	a	a	a	a

Sequential Pairwise Voting

First some ordering of alternatives is fixed. Then the first alternative in the ordering of alternatives is pitted against the second. The winning alternative (or both, if there is a tie) is then pitted against the third alternative in the ordering. An alternative is deleted whenever it loses. Those remaining at the end are declared to be the social choices.

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

For example, let the order be $a \to b \to c \to d \to e$. Then:

$a \to b \to c \to d \to e$

$b \to c \to d \to e$

$b \to d \to e$

$d \to e$

$A = \{d\}$

Dictatorship

First, one of the individuals is selected. Then we ignore all individual preference lists except that of the selected individual. The alternative on the top of the selected individual is declared to be the social choice.

V 1	V 2	V 3	V 4	V 5	V 6	V 7
a	a	a	c	c	b	e
b	d	d	b	d	c	c
c	b	b	d	b	d	d
d	e	e	e	a	a	b
e	c	c	a	e	e	a

If the selected individual is V 7, then obviously the social choice set is $\{e\}$.

Properties

The Pareto Condition:

If everyone prefers alternative x to alternative y, then y does not belong to the social choice set.

Monotonicity:

If alternative x belongs to the social choice set and someone moves x up one spot in his or her list, then x is still in the social choice set.

The Condorcet Winner Property:

An alternative x is called a *Condorcet winner* if, for every other alternative y, alternative x is above alternative y on strictly more than half of the lists. A procedure is said to satisfy the Condorcet winner property if the following is satisfied: If there exists a Condorcet winner, then it alone is the social choice.

Independence of Irrelevant Alternatives:

For every pair of alternatives x and y: If the social set includes x but not y, and one or more individuals change their preferences, but no one changes his or her preference between x and y, then the social choice still does not include y.

Sequential pairwise voting with a fixed agenda does not satisfy the Pareto condition.

Pareto condition: If everyone prefers alternative x to alternative y, then y is not a winner.

Voter 1	Voter 2	Voter 3
a	c	b
b	a	d
d	b	c
c	d	a

For example, let the order be $a \to b \to c \to d$. Then:

$a \to b \to c \to d$, $a \to c \to d$, $c \to d$, and d is a winner.

However, everyone prefers b to d.

	PAR	MON	CON	IND
PLU	yes	yes	no	no
BOR	yes	yes	no	no
HAR	yes	no	no	no
SEQ	no	yes	yes	no
DIC	yes	yes	no	yes

Sequential pairwise voting with a fixed agenda does not satisfy the Pareto condition:

Voter 1	Voter 2	Voter 3
a	c	b
b	a	d
d	b	c
c	d	a

The Hare procedure does not satisfy monotonicity:

V 1-7	V 8-12	V 13-16	V 17
a	c	b	b
b	a	c	a
c	b	a	c

V 1-7	V 8-12	V 13-16	V 17
a	c	b	a
b	a	c	b
c	b	a	c

The plurality voting does not satisfy the Condorcet winner condition:

V1	V2	V3	V4	V5	V6	V7	V8	V9
a	a	a	a	b	b	b	c	c
b	b	b	b	c	c	c	b	b
c	c	c	c	a	a	a	a	a

The Borda count does not satisfy the Condorcet winner condition:

V 1	V 2	V 3	V 4	V 5
a	a	a	b	b
b	b	b	c	c
c	c	c	a	a

The Hare procedure does not satisfy the Condorcet winner condition:

V 1	V 2	V 3	V 4	V 5
a	a	b	c	c
b	b	c	b	b
c	c	a	a	a

Dictatorship does not have the Condorcet winner property:

Voter 1	Voter 2	Voter 3
a	c	c
b	b	b
c	a	a

The plurality voting does not satisfy independence of irrelevant alternatives:

V1	V2	V3	V4	V1	V2	V3	V4
a	a	b	c	a	a	b	b
b	b	c	b	b	b	c	c
c	c	a	a	c	c	a	a

The Borda count does not satisfy independence of irrelevant alternatives:

V 1	V 2	V 3	V 4	V 5
a	a	a	c	c
b	b	b	b	b
c	c	c	a	a

V 1	V 2	V 3	V 4	V 5
a	a	a	b	b
b	b	b	c	c
c	c	c	a	a

The Hare procedure does not satisfy independence of irrelevant alternatives:

V1	V2	V3	V4	V1	V2	V3	V4
a	a	b	c	a	a	b	b
b	b	c	b	b	b	c	c
c	c	a	a	c	c	a	a

The sequential pairwise voting with does not satisfy independence of irrelevant alternatives:

V 1	V 2	V 3	V 1	V 2	V 3
c	a	b	b	a	b
b	c	a	c	c	a
a	b	c	a	b	c

Unofficial Answers

- Who cares.
- Whatever the origin of the problem might be, the problem itself is interesting and challenging.
- The problem was created as a by-product of our previous research.

Offical Answers

- The problem under consideration appears almost everywhere in Nature.

- It represents an important issue associated with many real world problems.

- Many interesting instances of the problem can be handled by our approach.