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Multiobjective De Novo Linear Programming

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CONTENTS

Introduction

- Linear programming (LP)– well-established discipline – set of feasible solutions- concept of optimal solutions
- Multi-objective linear programming (MOLP) –an extension of LP by multiple objectives – concept of non-dominated solutions (set) – a compromise solution
- LP and MOLP problems are models of given system – optimizing given systems by single or multiple objectives
- New concept of optimality: Optimizing given systems vs. designing optimal systems
- Multiobjective De Novo linear programming (MODNLP) – designing optimal systems by multiple objectives

Multiobjective linear programming problem (MOLP)

MOLP problem can be described as follows

$$\begin{aligned} \text{Max } \ & z = Cx \\ \text{s.t. } \ & Ax \leq b \\ & x \geq 0 \end{aligned} \qquad (1)$$

where

C is a (k, n) – matrix of objective coefficients,
A is a (m, n) – matrix of structural coefficients,
b is an m-vector of known resource restrictions,
x is an n-vector of decision variables,
z is a k-vector of objective values.

- In MOLP problems it is usually impossible to optimize all objectives together in a given system.
- Trade-off means that one cannot increase the level of satisfaction for an objective without decreasing this for another objective.
- Trade-offs are properties of inadequately designed system a thus can be eliminated through designing better one.

Solving of MOLP

- From optimal solutions to non-dominated solutions – compromise solution
- Two subjects - Decision Maker (DM) and Analyst (A)
- Classification of methods according to information mode:
  - With a priori information

    DM provides global preference information (weights, utility, goal values,..)

    A solves a single objective problem
  - With progressive information – interactive methods

    DM provides local preference information

    A solves local problems and provides current solutions
  - With a posteriori information

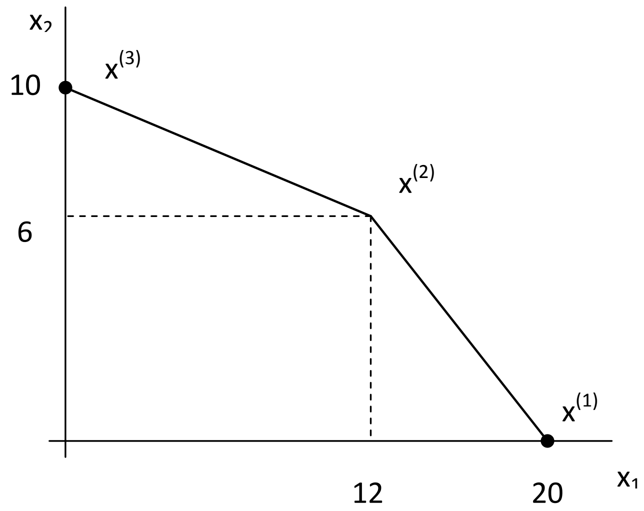
    A provides a non-dominated set

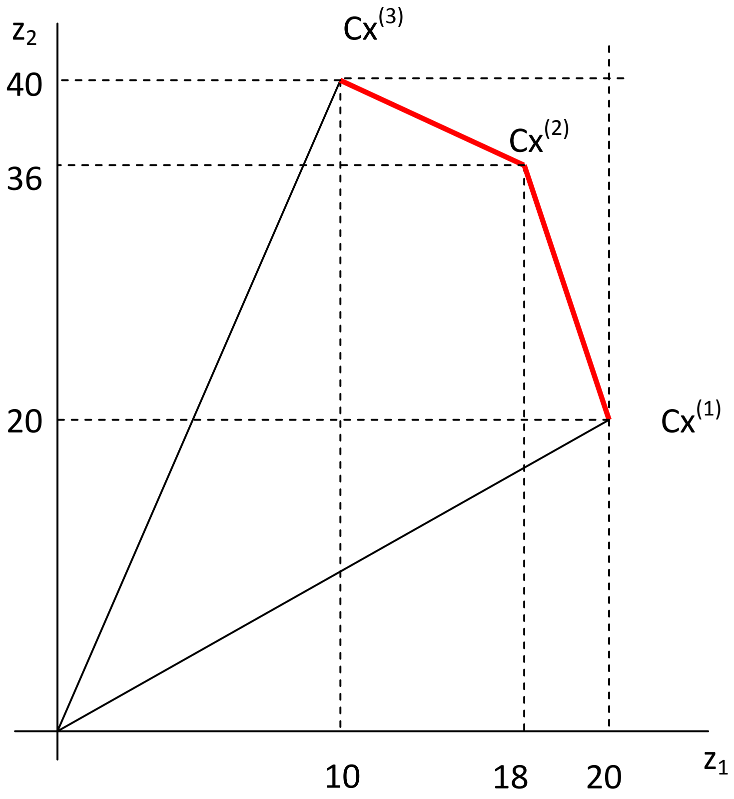
    DM provides global preference information on the non-dominated set

    A solves a single objective problem

Illustrative example

$$\begin{aligned} \max \quad & z_1 = x_1 + x_2 \\ \max \quad & z_2 = x_1 + 4x_2 \\ & 3x_1 + 4x_2 \le 60, \\ & x_1 + 3x_2 \le 30, \\ & x_1 \ge 0,\ x_2 \ge 0. \end{aligned}$$





Multiobjective De Novo linear programming problem (MODNLP)

Multiobjective De Novo linear programming (MODNLP) is problem for designing optimal system by reshaping the feasible set. By given prices of resources and the given budget the MOLP problem (1) is reformulated in the MODNLP problem (2)

$$\begin{aligned} \text{Max} \quad & z = Cx \\ \text{s.t.} \quad & Ax - b \leq 0 \\ & pb \leq B \\ & x \geq 0 \end{aligned} \qquad (2)$$

where

b is an m-vector of unknown resource restrictions,

p is an m-vector of resource prices, and

B is the given total available budget.

Solving of MODNLP

From (2) follows

$$pAx \leq pb \leq B$$

Defining n-vector of unit cost $v = pA$ we can rewrite problem (2) as

$$\begin{aligned} \text{Max} \quad & z = Cx \\ \text{s.t.} \quad & vx \leq B \\ & x \geq 0 \end{aligned} \qquad (3)$$

Solving single objective problems

$$\begin{aligned} \text{Max} \quad & z^i = c^i x \quad i = 1,2,\ldots,k \\ \text{s.t.} \quad & vx \leq B \\ & x \geq 0 \end{aligned} \qquad (4)$$

- z^* is k – vector of objective values for the ideal system with respect to B

MODNLP – metaoptimum problem

The metaoptimum problem can be formulated as follows

$$\begin{aligned} \text{Min} \quad & f = vx \\ \text{s.t.} \quad & Cx \geq z^* \\ & x \geq 0 \end{aligned} \qquad (5)$$

Solving problem (5) provides solution:

$$x^*$$
$$B^* = vx^*$$
$$b^* = Ax^*$$

The value B^* identifies the minimum budget to achieve z^* through x^* and b^*

Illustrative example – continued (1)

Input: $p = (5, 4)$ $B = 420$

Unit costs $v = pA = (19, 32)$

Max $z^i = c^i x$ $i = 1,2,...,k$

s.t. $vx \leq B$

$x \geq 0$

$z_1^* = 22{,}11$, $z_2^* = 52{,}50$

Min $f = vx$

s.t. $Cx \geq z^*$

$x \geq 0$

$x_1^* = 11{,}97$, $x_2^* = 10{,}13$

$B^* = vx^* = 551{,}71$

$b^* = Ax^*$ $b_1^* = 76{,}48$, $b_2^* = 42{,}39$

Optimum-path ratios

- The given budget level $B \leq B^*$
- The optimum -path ratio for achieving the best performance for a given budget B is defined as

$$r_1 = \frac{B}{B^*}$$

- Optimal system design for B : $x = r_1 x^*$, $b = r_1 b^*$, $z = r_1 z^*$
- The optimum-path ratio provides an effective and fast tool for the efficient optimal redesign of large-scale linear systems.
- There is possible define six types of optimum-path ratios (Shi, 1995):

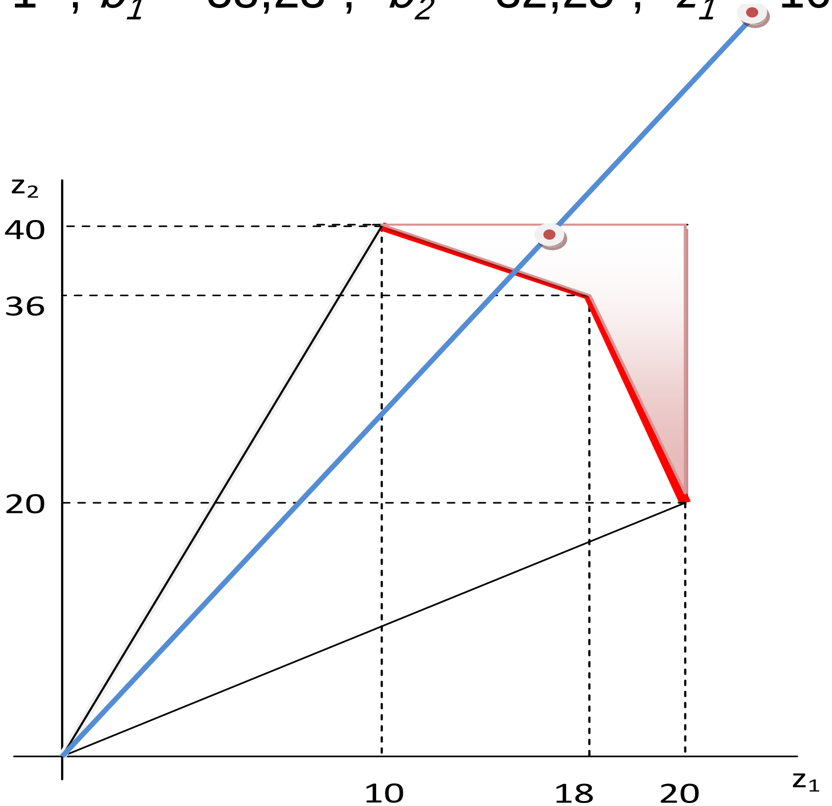
$$r_1 = \frac{B}{B^*} \qquad r_2 = \frac{B}{B^{**}} \qquad r_3 = \frac{B^*}{B^{**}}$$

$$r_4 = \frac{\sum_i \lambda_i B_i^j}{B} \qquad r_5 = \frac{\sum_i \lambda_i B_i^j}{B^*} \qquad r_6 = \frac{\sum_i \lambda_i B_i^j}{B^{**}}$$

Illustrative example – continued (2)

$r_1 = \frac{B}{B^*}$ = 0,761 Optimal system design for B : $x = r_1 x^*$, $b = r_1 b^*$, $z = r_1 z^*$

x_1 = 9,12 , x_2 = 7,71 , b_1 = 58,23 , b_2 = 32,25 , z_1 = 16,82 , z_2 = 39,96



Extensions

- Fuzzy De Novo Programming (FDNP) – fuzzy parameters, fuzzy goals, fuzzy relations, fuzzy approaches
- Interval De Novo programming (IDNP) - incorporating the interval programming and de Novo programming, allowing uncertainties represented as intervals within the optimization framework. The IDNP approach has the advantages in constructing optimal system design via an ideal system by introducing the flexibility toward the available resources in the system constraints.
- Complex types of objective functions -- the multiobjective form of Max (cx - pb) appears to be the right function to be maximized in a globally competitive economy.
- Searching for a better portfolio of resources - continuous reconfiguration and “reshaping” of systems boundaries.

Innovations

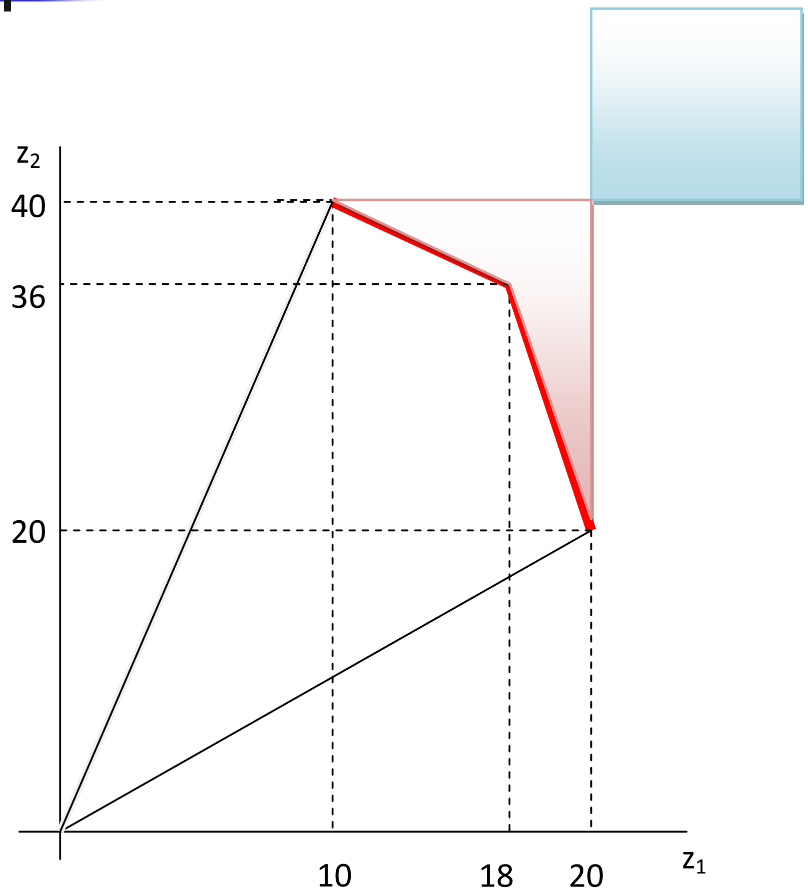
- Innovations bring improvements to the desired objectives and the better utilization of available resources
- The elements in the structural matrix A should be reduced by technological progress
- The technological innovation matrix
- T should be continuously explored

$$T = \begin{bmatrix} t_1 & 0 & \dots & 0 \\ 0 & t_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & t_m \end{bmatrix}$$

- The consumed rate of resource i : $0 \le t_i \le 1$

$$\begin{aligned} \text{Max} \quad & z = Cx \\ \text{s.t.} \quad & TAx - b \le 0 \\ & pb \le B \\ & x \ge 0 \end{aligned} \qquad (6)$$

Multiobjective optimization – dynamic process



short – term equilibrium:

trade-off

operational thinking

mid – term equilibrium:

trade-off free

tactical thinking

long – term equilibrium:

beyond trade-off free

strategic thinking

Illustrative example – continued (3)

Input: $p = (5, 4)$ $B = 420$, $t_1 = 0{,}8$, $t_2 = 0{,}7$

Unit costs $v = pTA = (14{,}8;\ 24{,}4)$

$z_1^* = 28{,}38$, $z_2^* = 68{,}85$

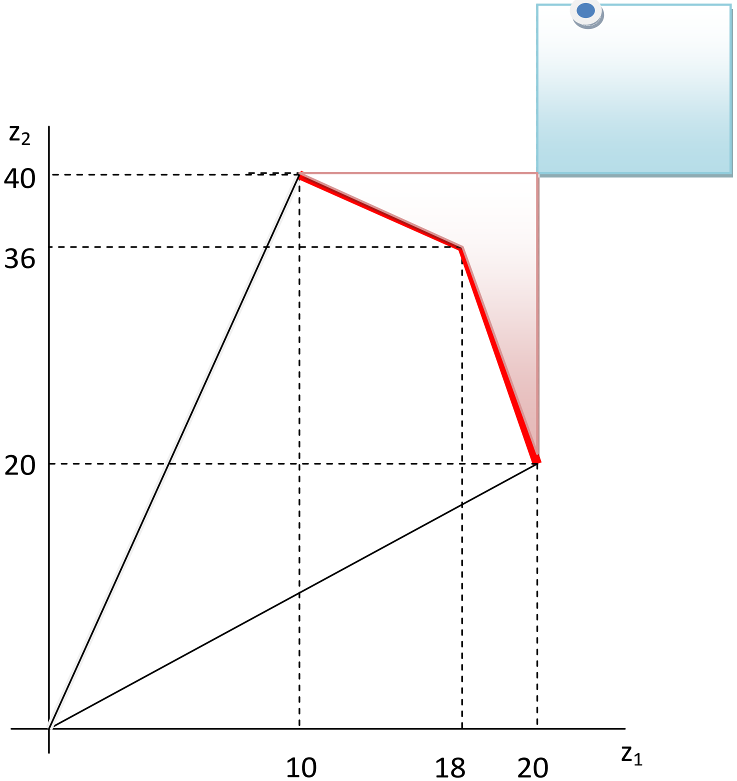
$x_1^* = 14{,}89$, $x_2^* = 13{,}49$

$B^* = vx^* = 549{,}52$

$$r_1 = \frac{B}{B^*} = 0{,}764$$

$x_1 = 11{,}38$, $x_2 = 10{,}31$

$z_1 = 21{,}69$, $z_2 = 52{,}62$



Applications - methodological

The tradeoffs-free decision making has a significant number of methodological applications – all such applications have the tradeoffs-free alternative in common:

- Compromise programming - minimize distance from the ideal point
- Risk management – portfolio selection - tradeoffs between investment returns and investment risk
- Game theory – win-win solutions
- Added value – value for the producer and value for the customer – both must benefit

Applications - real

Production planning

Babic, Z., Pavic, I.: Multicriterial production planning by De Novo programming approach. International Journal of Production Economics 43 (1996), 59-66.
Production plan for a real production system is defined taking into account financial constraints and given objective functions.

Water-resources-management

Zhang, Y. M., Huang, G. H., Zhang, X. D.: Inexact de Novo programming for water resources systems planning .European Journal of Operational Research 199 (2009), 531-541.

This paper presents an IDNP approach for the design of optimal water-resources-management systems under uncertainty. Optimal supplies of good-quality water are obtained in considering different revenue targets of municipal–industrial–agricultural competition under a given budget.

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Conclusions

- Traditional concepts of optimality focus on valuation of already given system.
- New cocepts of optimality – designing optimal systems
- The purpose is not to measure and evaluate tradeoffs, but to minimize or even eliminate them. An optimal system should be tradeoff-free.
- As a methodology of optimal system design we employ De novo programming for reshaping feasible sets in linear systems.
- Reformulation of MOLP problem by given prices of resources and the given budget
- Searching for metaoptimum with a budget B^*
- The optimum -path ratio for achieving the best performance for a given budget B
- Possible extensions of the concept
- Methodological and real applications